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LETTER TO THE EDITOR

Induced radiation of a charged particle in a time-periodic electromagnetic field

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Abstract. The radiation spectrum corresponding to transitions of a charge between quasi-energetic states is obtained. The induced radiation is found.

In considering quantum systems with time-periodic Hamiltonians the quasi-energy (QE) method proves to be the most effective. The concept of the four-dimensional quasi-momentum whose fourth component was called quasi-energy has been introduced by Nikishov and Ritus (1964) for an electron in the field of an intense wave. Zeldovitch (1966) and Ritus (1966) have introduced the concept of QE and quasi-energetic states (QES) for arbitrary time-periodic quantum systems. It should be noted that the radiation spectrum corresponding to transitions of an electron between the states coinciding with QES was first obtained by Blokhintsev (1933) in considering the Stark effect on hydrogen in a high-frequency electric field.

We shall calculate the induced dipole radiation of a particle with a charge $e (e > 0)$ and mass m moving in the time-periodic electromagnetic field. The Hamiltonian of this system is

$$H = (\boldsymbol{p} - e\boldsymbol{A})^2/2m, \qquad c = \hbar = 1, \qquad \boldsymbol{A}(t) = (\boldsymbol{\mathscr{H}}(t) \times \boldsymbol{r})/2, \tag{1}$$

where $\boldsymbol{p}$ is the operator of the particle momentum, $\boldsymbol{A}(t) = \boldsymbol{A}(t+T)$ is the time-periodic vector potential and $\boldsymbol{\mathscr{H}} = (0, 0, \mathscr{H})$ is the time-periodic magnetic field.

The Hamiltonian (1) may be presented as follows: $H = H_{x,y} + H_z$; $[H_{x,y}, H_z] = 0$. Here the Hamiltonian $H_{x,y}$ describes the motion in the x, y plane. The Hamiltonian H_z describes the free motion of a particle along the z axis with the momentum p_z.

For systems with the Hamiltonian $H_{x,y}$, when $\boldsymbol{A}(t)$ is an arbitrary function of time, Malkin and Man'ko (1970) have constructed four non-Hermitian integrals of motion $A, A^\dagger, B, B^\dagger$ obeying the boson commutation relations: $\mathrm{i}\,\partial A/\partial t = [H_{x,y}, A]$, $[A, A^\dagger] = 1$, $[B, B^\dagger] = 1$, $[A, B] = 0$. In the case of a time-periodic vector potential $\boldsymbol{A}(t)$ the eigenstates of the quadratic operators $A^\dagger A|n_1, n_2; t\rangle = n_1|n_1, n_2; t\rangle$; $B^\dagger B|n_1, n_2; t\rangle = n_2|n_1, n_2; t\rangle$ are QES (Malkin *et al* 1975):

$$|n_1, n_2; t+T\rangle = \exp\left(-\mathrm{i}T\sum_{j=1}^{2} (n_j + \tfrac{1}{2})\Omega_j\right)|n_1, n_2; t\rangle,$$

$$\Omega_{1;2} = \frac{1}{T}\int_0^T \left(\pm\frac{\Omega}{2} + |\epsilon|^{-2}\right) \mathrm{d}\tau; \qquad \Omega(t) = \frac{e\mathscr{H}(t)}{m}. \tag{2}$$

We shall consider the case of $\Omega_1 \pm \Omega_2 \neq s\omega$, $\omega = 2\pi/T$, $s = 0, \pm 1, \pm 2, \ldots$. The function $\epsilon(t)$ is defined by the relations

$$\ddot{\epsilon} + \Omega^2 \epsilon/4 = 0, \qquad \epsilon = |\epsilon| \exp\left(\mathrm{i}\int_0^t |\epsilon|^{-2}\, \mathrm{d}\tau\right),$$
$$\epsilon(t+T) = \exp\left[\mathrm{i}T(\Omega_1 + \Omega_2)/2\right]\epsilon(t), \tag{3}$$

the latter being valid only in a stability region. Below we shall consider that very case.

Using the scheme for calculating the radiation of non-stationary quadratic systems suggested by Ivanova *et al* (1974), let us express the operator $\boldsymbol{V} = (\boldsymbol{p} - e\boldsymbol{A})/m$ in terms of the boson operators $A, A^\dagger, B, B^\dagger$:

$$V_x = (\mathrm{i}\sigma_1 A + \sigma_2 B - \mathrm{i}\sigma_1^* A^\dagger + \sigma_2^* B^\dagger)/2\sqrt{m},$$
$$V_y = (\sigma_1 A + \mathrm{i}\sigma_2 B + \sigma_1^* A^\dagger - \mathrm{i}\sigma_2^* B^\dagger)/2\sqrt{m}, \tag{4}$$
$$\sigma_{1;2} = \left(\dot{\epsilon}^* \mp \mathrm{i} + \frac{\Omega}{2}\epsilon^*\right) \exp\left(\mp\int_0^t \frac{\Omega}{2}\, \mathrm{d}\tau\right).$$

Further, using the known formula obtained by Schwinger (1954) for the radiation power of a non-stationary system, we reduce the problem of calculating the induced radiation of the system (1) to calculating the Fourier decomposition of the matrix element of the operators (4).

When the system (1) transits between QES, two processes are possible: the first is the induced emission of a photon with the frequency $\omega_\lambda = \pm\Omega_j + \omega l > 0$, $j = 1, 2$, $l = 0, \pm 1, \pm 2, \ldots$; the second is the induced absorption of a photon with the frequency $\omega_\lambda = \pm\Omega_j + \omega l > 0$ (see Zeldovitch 1973).

Let us find the time-average dipole radiation power for the induced transition of the system (1) from the initial QES $|n_1, n_2; t\rangle$ to the final QES $|n_1 - 1, n_2; t\rangle$ or $|n_1, n_2 - 1; t\rangle$. Let an emitted photon have the frequency ω_λ, the wavevector $\boldsymbol{k}_\lambda$ and the polarization vector $\boldsymbol{e}_{\lambda,\sigma}$. Then the power radiated into a unit solid angle enclosing $\boldsymbol{k}_\lambda/\omega_\lambda$, and contained in a unit frequency interval about ω_λ, is

$$P(\boldsymbol{k}_\lambda, \omega_\lambda, \boldsymbol{e}_{\lambda,\sigma}) = \frac{\pi^2 e^2}{m\omega_\lambda} I_\sigma(\boldsymbol{k}_\lambda) n_j \sum_{l=-\infty}^{\infty} |\mu_j(l)|^2 \delta(\Omega_j - \omega l - \omega_\lambda), \tag{5}$$

where

$$\mu_j(l) = \frac{1}{T}\int_0^T \sigma_j(\tau) \exp[\mathrm{i}(\Omega_j - \omega l)\tau]\, \mathrm{d}\tau; \qquad j = 1, 2$$
$$(\boldsymbol{e}_{\lambda,\sigma})_z = 0; \qquad (\boldsymbol{e}_{\lambda,\sigma})_x^2 + (\boldsymbol{e}_{\lambda,\sigma})_y^2 = 1, \tag{6}$$

$I_\sigma(\boldsymbol{k}_\lambda)$ being the intensity of the incident radiation.

The induced absorption power for the transition from the initial QES $|n_1, n_2; t\rangle$ to the final QES $|n_1 - 1, n_2; t\rangle$ or $|n_1, n_2 - 1; t\rangle$ is obtained from (5) with the help of the substitution of $-\omega_\lambda$ for ω_λ. The induced radiation power for the transition from the initial QES $|n_1, n_2; t\rangle$ to the final QES $|n_1 + 1, n_2; t\rangle$ or $|n_1, n_2 + 1; t\rangle$ is obtained from (5) with the help of the substitution of $(n_j + 1)$ for n_j and $-\omega_\lambda$ for ω_λ. The induced absorption power for the transition from the initial QES $|n_1, n_2; t\rangle$ to the final QES $|n_1 + 1, n_2; t\rangle$ or $|n_1, n_2 + 1; t\rangle$ is obtained from (5) with the help of the substitution of $-(n_j + 1)$ for n_j.

Our main interest is to study the sum effect of the two simultaneous induced processes of both the emission and the absorption. The time-average power of these processes in the dipole approximation for the transition from the initial QES $|n_1, n_2; t\rangle$ to all final QES is

$$P(\boldsymbol{k}_\lambda, |\Omega_j - \omega l|, \boldsymbol{e}_{\lambda,\sigma}) = -(\pi^2 e^2/m) I_\sigma(\boldsymbol{k}_\lambda, \omega_\lambda = |\Omega_j - \omega l|)|\mu_j(l)|^2/(\Omega_j - \omega l). \quad (7)$$

From (7) it is obvious that in the case of $\Omega_2 < 0$ the sum effect of the two simultaneous induced processes with the basic frequency $|\Omega_2|$ is the induced radiation. Further, the effect of the two simultaneous induced processes with frequency $|\Omega_j - \omega l|$ is the induced radiation if the number of the satellite l is selected in accord with the condition $\Omega_j - \omega l < 0$.

It is also necessary to emphasize that all conclusions concerning the induced radiation of a charge in the time-periodic field (1) are entirely transferred to any other time-periodic $3N$-dimensional system with the discrete spectrum of QE whose Hamiltonian is a quadratic form with respect to coordinates and momenta.

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